Recall: Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \longrightarrow \sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\sec^2 x = 1 + \tan^2 x \longrightarrow \tan^2 x = \sec^2 x - 1$$

Double Angle (Reduction) Formulas

$$\sin 2x = 2\sin x \cos x \longrightarrow \frac{1}{2}\sin 2x = \sin x \cos x$$

$$\cos 2x = 2\cos^2 x - 1 \longrightarrow \frac{\cos 2x + 1}{2} = \cos^2 x = \boxed{\frac{1}{2}(\cos 2x + 1)}$$

$$\cos 2x = 1 - 2\sin^2 x \longrightarrow \frac{\cos 2x - 1}{-2} = \frac{-\cancel{2}\sin^2 x}{-\cancel{2}}$$

solve for $\sin^2 x$

$$\sin^3 x = -\frac{(\cos 2x - 1)}{2}$$

$$= \frac{-\cos 2x + 1}{2} \text{ OR } \boxed{\frac{1}{2}(1 - \cos 2x) = \sin^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

ex. $\int \cos^3 x\,dx = \int \cos^2 x \overset{du}{\cos x\,dx}$

write ITO $\sin x$

$\cos^2 x = 1 - \sin^2 x$

$$= \int (1 - \sin^2 x)\cos x\,dx$$

$u = \sin x$
$du = \cos x\,dx$

$$= \int (1 - u^2)\,du$$

$$= u - \frac{u^3}{3} + C$$

$$= \boxed{\sin x - \frac{1}{3}\sin^3 x + C}$$

recall $\int \sin x \cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$$= \int u\,du$$

$$= \frac{u^2}{2} + C = \boxed{\frac{1}{2}\sin^2 x + C}$$

$$\boxed{\sin^2 x = \frac{1}{2}(1-\cos 2x)}$$

ex. $\int_0^{\pi} \sin^2 x\, dx = \frac{1}{2}\int_0^{\pi}(1-\cos 2x)\, dx$

$$= \frac{1}{2}\int_0^{\pi} 1\, dx - \frac{1}{2}\int_0^{\pi} \cos \overset{u}{2}x\, dx$$

$$= \frac{1}{2}x\Big|_0^{\pi} - \frac{1}{2}\cdot\frac{1}{2}\sin 2x\Big|_0^{\pi}$$

$$= \frac{1}{2}(\pi - 0) - \underbrace{\frac{1}{4}\left(\cancel{\sin 2\pi}^{0} - \cancel{\sin 0}^{0}\right)}_{0}$$

$$= \frac{1}{2}\pi$$

$$= \boxed{\frac{\pi}{2}}$$

$\int \sin 2x\, dx$

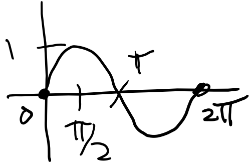
$= \frac{1}{2}\int \sin u\, du$

$u = 2x$

$du = 2dx$

$\Downarrow$

$\frac{1}{2}du = dx$



$y = \sin x$

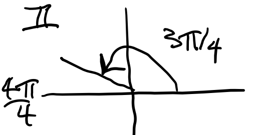
ex. $\int_{\pi/2}^{3\pi/4} \sin^5 x \underbrace{\cos^3 x}\, dx = \int_{\pi/2}^{3\pi/4} \sin^5 x\, \underbrace{\cos^2 x}\, \underbrace{\cos x\ dx}_{du}$

↑ write ITO sinx

$\cos^2 x = 1 - \sin^2 x$

$$= \int_{\pi/2}^{3\pi/4} \sin^5 x(1-\sin^2 x)\cos x\, dx$$

know $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$



ASTC sin ⊕ in QII

$$= \int_1^{\sqrt{2}/2} u^5(1-u^2)\, du$$

↪ distribute to get a polynomial

$u = \sin x$

$du = \cos x\, dx$

$u_a = \sin\frac{\pi}{2} = 1$

$u_b = \sin\frac{3\pi}{4}$

$= \sin\pi/4 = \sqrt{2}/2$

$$= \int_1^{\sqrt{2}/2}(u^5 - u^7)\, du$$

$$= \left(\frac{u^6}{6} - \frac{u^8}{8}\right)\Bigg|_1^{\sqrt{2}/2}$$

$$= \frac{1}{6}\left(\left(\frac{\sqrt{2}}{2}\right)^6 - 1^6\right) - \frac{1}{8}\left(\left(\frac{\sqrt{2}}{2}\right)^8 - 1^8\right)$$

$\sqrt{2} = 2^{1/2}$

$$= \frac{1}{6}\left(\frac{(2^{1/2})^6}{2^6} - 1\right) - \frac{1}{8}\left(\frac{(2^{1/2})^8}{2^8} - 1\right)$$

$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

$$= \frac{1}{6}\left(\frac{2^3}{2^6} - 1\right) - \frac{1}{8}\left(\frac{2^4}{2^8} - 1\right)$$

$(x^a)^b = x^{ab}$

$$= \frac{1}{6}\left(\frac{1}{2^3} - 1\right) - \frac{1}{8}\left(\frac{1}{2^4} - 1\right)$$

$$= \frac{1}{6}\left(\frac{1}{8} - \frac{8}{8}\right) - \frac{1}{8}\left(\frac{1}{16} - \frac{16}{16}\right)$$

$$= \frac{1}{6}\left(-\frac{7}{8}\right) - \frac{1}{8}\left(-\frac{15}{16}\right)$$

$\vdots$

$$= \boxed{-\frac{11}{384}}$$

$a^2 b^2 = (ab)^2$

ex. $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx = \int_0^{\pi/2} (\sin x \cos x)^2 \, dx$

$$= \int_0^{\pi/2} \left(\frac{1}{2}\sin 2x\right)^2 dx$$

$$= \int_0^{\pi/2} \left(\frac{1}{2}\right)^2 (\sin 2x)^2 \, dx$$

$$= \frac{1}{4}\int_0^{\pi/2} \sin^2(2x) \, dx$$

$$= \frac{1}{4}\cdot\frac{1}{2}\int_0^{\pi/2} (1-\cos 4x) \, dx$$

$$= \frac{1}{8}\left(x - \frac{1}{4}\sin 4x\right)\Big|_0^{\pi/2}$$

$$= \frac{1}{8}\left[\left(\frac{\pi}{2}-0\right) - \frac{1}{4}\left(\sin\left(4\cdot\frac{\pi}{2}\right) - \sin 4(0)\right)\right]$$

$$= \frac{1}{8}\left[\left(\frac{\pi}{2}\right) \underbrace{- \frac{1}{4}\left(\cancel{\sin 2\pi}\xrightarrow{} 0 - \cancel{\sin 0}\xrightarrow{} 0\right)}_{0}\right]$$

$$= \boxed{\frac{\pi}{16}}$$

$$\boxed{\frac{1}{2}\sin 2x = \sin x \cos x}$$

$$\cos 2\overset{2x}{u} = 1 - 2\sin^2 \overset{2x}{u}$$

$$\cos(4x) = 1 - 2\sin^2(2x)$$

$$\frac{1}{2}(1-\cos 4x) = \sin^2(2x)$$

Compare and Contrast:

ex. $\int \underline{\sec^3 x}\tan x\,dx = \int \underline{\sec^2 x \cdot} \overbrace{\underline{\sec x}\tan x\,dx}^{du}$

$u = \sec x$
$du = \sec x \tan x\,dx$

$= \int u^2\,du$

$= \frac{u^3}{3} + C$

$= \boxed{\frac{1}{3}\sec^3 x + C}$

ex. $\int \tan^3 x \sec x\,dx = \int \tan^2 x \overbrace{\tan x \sec x\,dx}^{du}$

↑ write ITO secx

$\sec^2 x = 1 + \tan^2 x$
$\therefore \tan^2 x = \sec^2 x - 1$

$= \int (\sec^2 x - 1)\sec x \tan x\,dx$

$u = \sec x$
$du = \sec x \tan x\,dx$

$= \int (u^2 - 1)\,du$

$= \frac{u^3}{3} - u + C$

$= \boxed{\frac{1}{3}\sec^3 x - \sec x + C}$

ex. $\int \frac{dx}{x^2\sqrt{1-x^2}}$ write ITO θ

$x = \sin\theta$ $dx = \cos\theta \, d\theta$

$x^2 = \sin^2\theta$

$= \int \frac{\cos\theta \, d\theta}{\sin^2\theta \sqrt{1-\sin^2\theta}}$ simplify

$1-\sin^2\theta = \cos^2\theta$

$\therefore \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \sqrt{\cos\theta^{\cancel{2}}}$

$= \int \frac{\cancel{\cos\theta}}{\sin^2\theta \ \cancel{\cos\theta}} \, d\theta$

$= \int \frac{1}{\sin^2\theta} \, d\theta$

$\frac{1}{\sin\theta} = \csc\theta$

$= \int \csc^2\theta \, d\theta$

$(\cot\theta)' = -\csc^2\theta$

$\therefore -(\cot\theta)' = \csc^2\theta$

$= -\cot\theta + C$

↑ convert back to x

$= -\frac{\cos\theta}{\sin\theta} + C$

$\sin\theta = \frac{x}{1} \quad \frac{O}{H}$

$x^2 + A^2 = 1^2$

$x^2 + A^2 = 1$

$A = \sqrt{1-x^2}$

$= -\frac{\cos\theta}{x} + C$

$\cos\theta = \frac{A}{H} = \frac{\sqrt{1-x^2}}{1}$

$\cos\theta = \sqrt{1-x^2}$

$= \boxed{-\frac{\sqrt{1-x^2}}{x} + C}$ ☺

ex. $\int \frac{dx}{x^2\sqrt{4-x^2}}$

next time